Association of Coaching Institutes

Redefining Education

Paper Contributor : SNEHA Tuition Classes

Time : 3:00 Hrs. **Max. Marks : 80**

General Instructions:

1. Question paper consists of 34 questions divided into Four Sections, namely A, B, C and D.
   (i) Section –A :Q. No 1 contains 8 multiple choice type of questions carrying two marks each.
   Q. No. 2 Contains 4 very short answer type of questions carrying one mark each.
   (ii) Section – B : Q. No. 3 to Q.14 are 12 questions carrying two marks each.
   (iii) Section – C : Q. No. 15 to Q. No.26 are 12 questions carrying Three marks each
   (iv) Section –D : Q. No. 27 to Q.No.34 are 8 questions carrying four marks each.
2. Figures to the right indicate full marks.
3. Start each section on new page.
4. For each MCQ, the correct answer must be written along with it's alphabet.
   e.g. (a)..../(b)......./(c)....../(d)...... etc
5. Evaluation of each MCQ would be done for the first attempt only.
6. Use of graph paper is not necessary. Only rough sketch is expected.
7. Use log table if necessary. Use of calculator is not allowed.

SECTION - A

Q.1 Select and write the correct answer. **(16 Marks)**

1. Inverse of statement pattern $(p \vee q) \rightarrow (p \wedge q)$ is

(a) $(p \wedge q) \rightarrow (p \vee q)$ (b) $\sim (p \vee q) \rightarrow (p \wedge q)$

(c) $(\sim p \vee \sim q) \rightarrow (\sim p \wedge \sim q)$ (d) $(\sim p \wedge \sim q) \rightarrow (\sim p \vee \sim q)$

2. The general solution of $\tan x = -1$ is

(a) $x = n\pi + \frac{\pi}{4}, n \in z$ (b) $x = n\pi \pm \frac{3\pi}{4}, n \in z$ (c) $x = n\pi + \frac{3\pi}{4}, n \in z$ (d) $x = \frac{n\pi}{2} + \frac{3\pi}{4}, n \in z$

3. If the vectors $2\hat{i} - q\hat{j} + 3\hat{k}$ and $4i - 5j + 6k$ are collinear, then value of q is _______

(a)5 (b) 10 (c) 5/2 (d) 5/4

4. The direction ratios of two perpendicular lines are $k, -6, -2$ and $(k-1), k, 4$ then values of k are

(a) $8, -1$ (b) 2,3 (c) 8,1 (d) $-8, -1$

5. If $y = 1 - \cos\theta$, $x = 1 - \sin\theta$ then $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ is

(a) -1 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

6. $\int \frac{\sin 3x}{\sin x} dx = -----$

(a) $x - \sin 2x + c$ (b) $x + \sin 2x + c$ (c) $x + \cos 2x + c$ (d) $x - \cos 2x + c$

7. Function $f(x) = x^2 - 3x + 4$ has minimum value at $x = ---$

(a)0 (b) $\frac{-3}{2}$ (c) 1 (d) 3/2

8. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^3\right]^{\frac{7}{3}} = 7\left(\frac{d^2y}{dx^2}\right)$ are

(a) 2,3 (b) 3,2 (c) 2,2 (d) 3,3

Q.2 Answer the following **(04 Marks)**

(i) Write the negation of

All students are sincere

(ii) Find the principle value of $\sin^{-1}\left(\frac{1}{2}\right)$

(iii) Find $\frac{dy}{dx}$ if $y = \sqrt{\sin x}$

(iv) Write $\int \frac{1}{\sqrt{x^2+a^2}}dx =$

SECTION – B

Attempt Any Eight of the following **(16 Marks)**

Q.3 Write the truth values of the following

(i) $\sqrt{5}$ is irrational but $3+\sqrt{5}$ is a complex number (ii) If ABC is a triangle and all it's sides are equal than each angle has measure 30^0

Q.4 Find the polar coordinates of the point whose cartesian coordinates are (1, $\sqrt{3}$)

Q.5 Find k, if the sum of slopes of the lines represented by the equation $x^2 + kxy - 3y^2 = 0$ is twice their product.

Q.6 The cartesian equation of a line is $\frac{x-6}{2} = \frac{y+4}{7} = \frac{z-5}{3}$, find the vector equation of the line.

Q.7 Find the Cartesian equation of the line passing through the points $A(3,4,-7)$ and $B(6,-1,1)$

Q.8 Find the vector equation of the plane passing through a point having position vector $3i - 2j + k$ and perpendicular to the vector $4i + 3j + 2k$.

Q.9 If f(x) is continuous at $x = 0$, where

$$f(x) = \frac{1-\cos kx}{x^2}, \quad \text{for } x \neq 0$$
$$= \frac{1}{2}, \quad \text{for } x = 0$$

Then find the value of k

Q.10 Find $\frac{dy}{dx}$ if $y = \sin^{-1}(2\cos^2 x - 1)$

Q.11 If $\sec(\frac{x+y}{x-y}) = a^2$, show that $\frac{dy}{dx} = \frac{y}{x}$

Q.12 Evaluate $\int_{-\pi/4}^{\pi/4} \frac{1}{1+\sin x}dx$

Q.13 Three balanced coins are tossed simultaneously. If x denotes the no. of heads, find probability distribution of x.

Q.14 Given $x \sim B(n,p)$, If $n = 20, E(x) = 10$ find p and var(x)

SECTION- C

Attempt Any Eight of the following **(24 Marks)**

Q.15 Determine whether the following statement pattern is a tautology or a contradiction or contingency $(p \wedge q) \vee (p \wedge r)$

Q.16 If $\tan^{-1} 2x + \tan^{-1} 3x = \pi / 4$ then find the value of x

Q.17 Find the vector equation and cartesian equation of a line passing through the points A(3,4,-7) and B(6,7,1)

Q.18 If l, m, n are the direction cosines of a line, then prove that $l^2 + m^2 + n^2 = 1$

Q.19 Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

Q.20 Find the equation of the planes parallel to the plane $x + 2y + 2z + 8 = 0$. Which are at a distance of 2 units from the point $(1,1,2)$

Q.21 If x and y are differentiable functions of t, then show that $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, if $\frac{dx}{dt} \neq 0$

Q.22 Evaluate: $\int \frac{1}{\cos(x-a)\cos(x-b)}.dx$

Q.23 Evaluate: $\int_0^{\pi/2} \frac{1}{5+4\cos x}.dx$

Q.24 Find the area of the sector of the circle bounded by $x^2 + y^2 = 16$ and the line $y = x$ in the first quadrant.

Q.25 A fair coin is tossed 3 times. Let x be the number of heads obtained. Find $E(x)$ *and* $\text{var}(x)$.

Q.26 Let the p.m.f of r.v.x be

$$p(x) = \binom{4}{x}\left(\frac{5}{9}\right)^x \left(\frac{4}{9}\right)^{4-x}, x = 0,1,2,3,4$$

Find E(x) and var(x)

SECTION- D

Attempt Any Eight of the following **(20 Marks)**

Q.27 Discuss the continuity of the following functions. If the function have a removable discontinuity redefine the function so as to remove the discontinuity

$$\begin{aligned} f(x) &= \frac{4^x - e^x}{6^x - 1}, \quad \text{for } x \neq 0 \\ &= \log(2/3), \quad \text{for } x = 0, \text{ test at } x = 0 \end{aligned}$$

Q.28 Show that $\cos^{-1}(4/5) + \cos^{-1}(12/13) = \cos^{-1}\left(\frac{33}{65}\right)$

Q.29 Show that the acute angle θ between the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$ is given by

$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$, if $a + b \neq 0$. Find the condition, if the line are parallel.

Q.30 Minimize: $z = 6x + 2y$, subject to $5x + 9y \leq 90$, $x + y \geq 4$, $y \leq 8$, $x \geq 0$, $y \geq 0$,

Q.31 A function f(x) is defined as

$$\begin{aligned} f(x) &= x + a, \quad \text{for } x < 0 \\ &= x \quad , \quad \text{for } 0 \leq x < 1 \\ &= b - x, \quad \text{for } x \geq 1 \end{aligned}$$

Is continuous on it's domain. Find $(a + b)$

Q.32 If υ and v are functions of x, then $\int u\,v.dx = u\int v.dx - \int\left[\frac{d}{dx}u\int v.dx\right]dx$. Hence Evaluate $\int xe^{x}\,dx$

Q.33 Find the particular solution of the different equation $\cos(x+y)dy = dx$,when $x = 0$ and $y = 0$

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